9. The Darmstadt Slide Rule and the P-Scale,

door: John Mosand, Norway

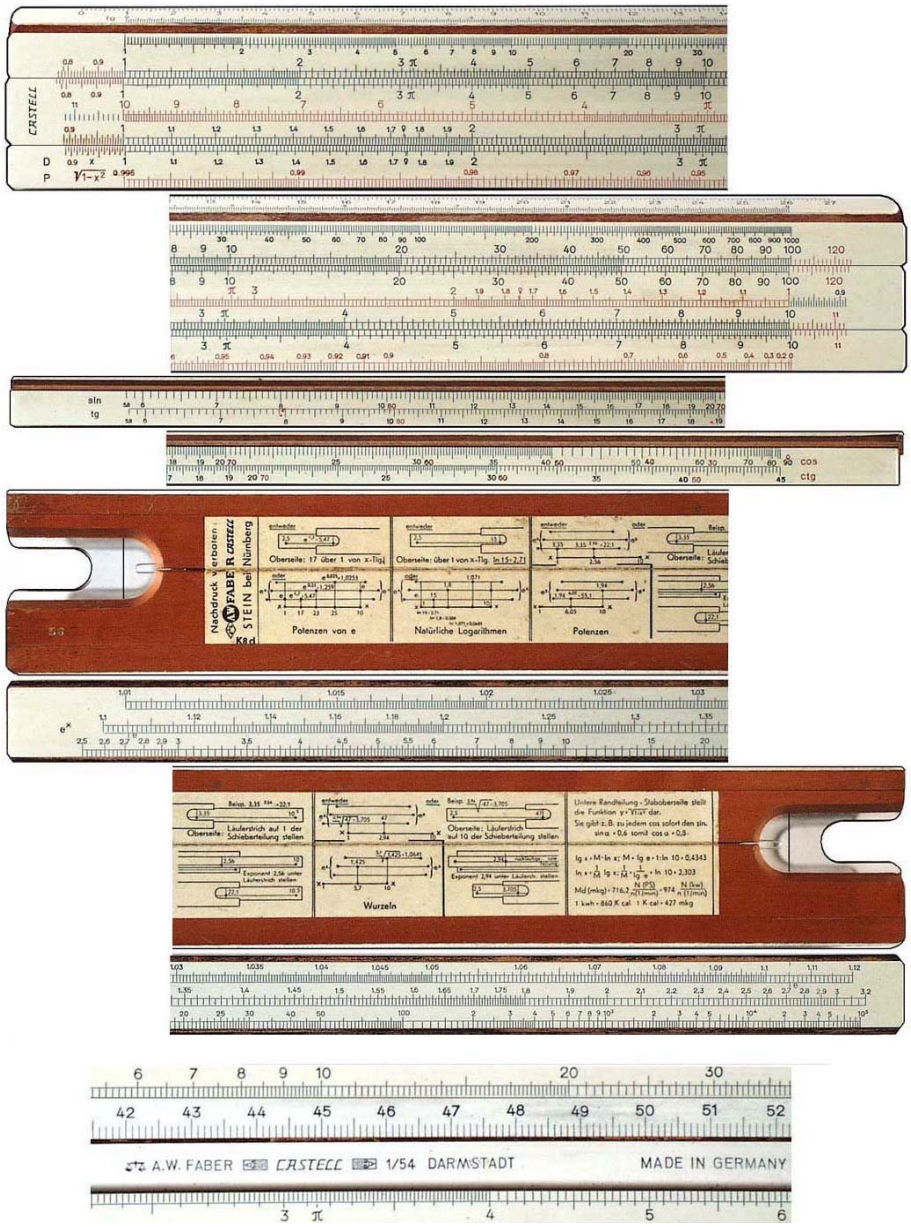
The Mannheim and Darmstadt slide rules
The slide rule model developed by Amedee Mannheim around 1850 can be argued to be the first modern slide rule, in that it established a basic shape and a basic layout of scales that became a pattern for further developments.

The next important step in the direction of the Darmstadt model was the scale layout of Max Rietz from 1902.[1] Rietz supplemented the basic Mannheim model with a third-power scale (K) and a log scale (L). Later this was extended with an ST scale and an inverted C scale (CI). As we know, the Rietz system became very popular and was produced in enormous quantities until the end of the slide rule era.

1934/35 saw the introduction of our present subject, the Darmstadt System, first marketed by Faber-Castell. Professor Walther of the Darmstadt Technische Hochschule added three LL scales and a 'Pythagorean' scale (P) to the Rietz layout.[2] The LL scales were located on the back of the slide, where the trigonometric scales had been on the Rietz. These were moved to the front or, at first, to the edges of the rule.

The three LL scales for positive exponents to the constant 'e' greatly simplified calculations involving non-integer powers. Negative exponents could be obtained by using inverses.
It can be mentioned that the Japanese Yokota patented a slide rule with three 'positive' and three 'negative' LL scales already in 1907.[3]

The Pythagorian Scale



The most characteristic scale on the Darmstadt rule is the Pythagorean one. This P-scale is quite versatile and can be used for a number of applications:

(1) Calculations involving right triangles.
(2) Reading cosine values directly from sine values, and vice versa.
(3) Calculations for certain electrical circuits.
(4) Finding orthogonal vectors from an angled vector.
(5) Finding more accurate values for sines to very large angles and cosines to very small angles.

... and other applications which the user might find convenient.

It is quite striking and puzzling that although the Darmstadt found a popular market among professionals over most of the world, it never seemed to catch on in the US. None of the popular American slide rules had a P scale.

Possibly the most famous of all Darmstadts is the F-C 1/54 which was made from 1935 until the end in 1976 with very few changes. From 1950 they also made an alternative all plastic version, the 111/54.

Interestingly, the Darmstadt was never patented, so all the major European and Japanese manufacturers made at least one version of the Darmstadt.

It is no surprise that this concept was developed further, with additional scales. E.g. the Aristo BiScholarLL retained the three LL scales but added several inverted scales and two tangent scales, while their most developed rule, the HyperLog, has eight LL scales and six hyperbolic scales. Clearly, a duplex layout with all scales on the surface is more convenient than having the LL scales on the back of the slide.

The Aristo MultiTrig has the P function distributed over two scale lengths. The additional P scale relates to the ST scale rather than the S scale. (Lacking LL scales, it is not a Darmstadt rule.) As an aside: regular slide rules are often said to be generally limited to an accuracy of three digits. The MultiTrig P-scales give an accuracy up to six digits.

The almost 'mysterious' F-C Mathema also has two P scales, one regular with a negative and one with a positive last term under the root.

The always popular Darmstadt models were naturally made in all the standard sizes, 12.5 cm scales, 25 cm and 50 cm, as well as in demonstrator versions.

The layout and sequence of scales can vary considerably from one manufacturer to another, and some times the term 'Darmstadt' isn't even mentioned. Dr. Günter Kugel, who is perhaps the most prominent authority on the subject, says that a true Darmstadt must have at least the scales found on the original model. How many additional scales it can have and still be a Darmstadt isn't defined. Personally, I would limit it to a few inverted scales, including a BI scale, and additional trig scales – and call it a "Modified Darmstadt Type" slide rule.

Dr. Kugel also wrote a comprehensive article on the F-C Darmstadts, published in the periodical "Historische Bürowelt" in issues no. 47 and 48, 1997.

Footnotes:
[1] 100 Jahre ARISTO-Rechenstäbe. Hamburg, 1972.
[2] Dr. Günter Kugel: Darmstadt-Rechenschieber. Lecture 2001, RS Brief 1-2002.
[3] Hans Dennert: D&P and Aristo Slide Rules 1872-1978. OSJ Vol.6, No.1, 1997.

Some Typical Calculations on a Darmstadt
(1) Finding the value for LL with a negative exponent:
Find the value for the corresponding positive exponent. Transfer this to the C scale. Read the answer on the CI scale.

(2) For a right triangle with unit hypotenuse and one side known, find the other side:
Set hairline at the length of the known side on D scale. Read length of unknown side on P scale.

(3) From a known sine value, find the cosine:
Set hairline at sine value on D scale. Read cosine on P scale. For finding sine from a known cosine, use the reverse procedure.

(4) Without a P scale, it is difficult to get accurate values for sines of very large angles and cosines of very small angles. In such cases, use sines for cosines and cosines for sines and read answers on the P scale at a greater accuracy.

(5) To find the orthogonal components of an angled vector, think of the vector as the hypotenuse (h) in a right triangle: set ‘h’ on C over an index. Move cursor to cos of the angle. Read length of the one side on D. Read value on P. Move cursor to this value on D. Read length of the other side on C.

About the Author
John Mosand, Trondheim, Norway. Born in 1935. Retired architect. Also former professional oboist in the Trondheim Symph. Orch. Many interests in the arts and sciences.